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An infinite plate loaded with a normal force moving along a complex open trajectory

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Introduction. A method for solving the problem on the action of a normal force moving on an infinite plate according to an arbitrary law is considered. This method and the results obtained can be used to study the effect of a moving load on various structures.

Materials and Methods. An original method for solving problems of the action of a normal force moving arbitrarily along a freeform open curve on an infinite plate resting on an elastic base, is developed. For this purpose, a fundamental solution to the differential equation of the dynamics of a plate resting on an elastic base is used. It is assumed that the movement of force begins at a sufficiently distant moment in time. Therefore, there are no initial conditions in this formulation of the problem. When determining the fundamental solution, the Fourier transform is performed in time. When the Fourier transform is inverted, the image is expanded in terms of the transformation parameter into a series in Hermite polynomials.

Results. The solution to the problem on an infinite plate resting on an elastic base, along which a concentrated force moves at a variable speed, is presented. A smooth open curve, consisting of straight lines and arcs of circles, was considered as a trajectory. The behavior of the components of the displacement vector and the stress tensor at the location of the moving force is studied, as well as the process of wave energy propagation, for which the change in the Umov-Poynting energy flux density vector is considered. The effect of the speed and acceleration of the force movement on the displacements, stresses and propagation of elastic waves is investigated. The influence of the force trajectory shape on the stress-strain state of the plate and on the nature of the propagation of elastic waves is studied. The results indicate that the method is quite stable within a wide range of changes in the speed of force movement.

Discussion and Conclusions. The calculations have shown that the most significant factor affecting the stress-strain states of the plate and the propagation of elastic wave energy near the concentrated force is the speed of its movement. These results will be useful under studying dynamic processes generated by a moving load.

Keywords: infinite plate, moving load, arbitrary open trajectory, variable speed, energy of elastic waves.

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Introduction. The regularities of dynamic processes in solid media caused by the action of a moving load are of considerable interest, and solutions to such problems find numerous applications and involve the use of various methods. In a number of works, to exclude time from the number of independent variables, a mobile coordinate system was introduced [1−2] or a quasistatic formulation of the problem was considered [3−6]. The finite element method [7], variational methods [8−10], as well as direct methods [11−13] proved to be quite effective in solving these problems. In [14−15], the method of boundary integral equations was used, and in [16] — a method based on the application of fundamental solutions to the corresponding differential equations. In this paper, the given method is used to solve the problem of the action of a normal force moving along a freeform open curve on an infinite plate resting on an elastic base.

Problem statement. Following [17, 18], this problem is reduced to solving the equation:

$$\Delta^2 U + c^{-2}\partial_t^2 U + kU = \frac{P}{D}, \tag{1}$$

where U — the plate deflection; $D = \frac{EH^3}{12(1-\mu^2)}$; E — the Young's modulus; μ — the Poisson's ratio; H — the plate thickness; $c^{-2} = \frac{\rho H}{D}$; ρ — the density of the plate material; $k = \frac{k_o}{D}$; k_o — the stiffness coefficient of the elastic base.

The solution to this equation corresponds to the energy flow directed from the excitation sources to infinity. We will assume:

$$P = \delta(x - x_o(t))\delta(y - y_o(t)).$$

This force moves along an open trajectory γ, whose beginning and end go to infinity. The parametric setting of the trajectory has the form: $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$, where t — time. It is assumed that the force starts to move at the beginning of the trajectory, located at a sufficient distance from the place where its effect on the plate is being studied at the moment of time $t = -\infty$. Therefore, there are no initial conditions in such a statement.

Materials and Methods. Consider the fundamental solution to equation (1), which can be obtained from the equation:

$$\Delta^2 W + c^{-2}\partial_t^2 W + kW = \frac{1}{D}\delta(x - x_o)\delta(y - y_o)\delta(t - \tau). \tag{2}$$

It is known that the solution to equation (1) can be presented as:

$$U(x, y, t) = \int_{-\infty}^{\infty}\iint_{R^2} W(x, x_o, y, y_o, t - \tau)P(x_o, y_o, \tau)dx_o dy_o d\tau.$$

In our case, taking into account a specific type of moving force, we have:

$$U(x, y, t) = \int_{-\infty}^{\infty} W(x, x_o(\tau), y, y_o(\tau), t - \tau)d\tau.$$

Applying the Fourier transform in time to equation (2), we obtain the differential equation:

$$\Delta^2 W_o - \omega^2 c^{-2} W_o + kW_o = \frac{1}{D}\delta(x - x_o)\delta(y - y_o)e^{i\omega\tau}. \tag{3}$$

Using the limiting absorption principle and the Fourier transform with respect to variables *x* and *y*, and under the condition $k > \frac{\omega^2}{c^2}$ we can obtain a solution to the equation (3):

$$W_0\left(x, x_0 y, y_0, \frac{\omega^2}{c^2}\right) = \frac{i}{4\pi\chi^2 D}[K_0(\alpha_1 R) - K_0(\alpha_2 R)],$$

where $R = \left[(x - x_0)^2 + (y - y_0)^2\right]^{1/2}$; $\chi = \sqrt[4]{k - \omega^2/c^2}$; $\alpha_1 = \chi e^{i\pi/4}$; $\alpha_2 = \chi e^{-i\pi/4}$; $K_0(z)$ — the Macdonald function.

Under the condition $k \le \frac{\omega^2}{c^2}$ the solution to equation (3) looks like this:

$$W_0(x, x_0, y, y_0, \omega^2/c^2) = \frac{i}{4\pi\chi^2 D}\left[\frac{\pi i}{2} H_0^{(1)}(\chi R) - K_0(\chi R)\right],$$

where $\chi = \sqrt[4]{\frac{\omega^2}{c^2} - k}$; $H_0^{(1)}(z)$ — the Hankel function.

To reverse the Fourier transform, the solution $W_0(x, x_0, y, y_0, \omega^2/c^2)$ is expanded by variable $\frac{\omega}{c}$ into series according to the system of orthogonal functions $\left\{e^{-\omega^2/c^2} H_k\left(\frac{\omega}{c}\right)\right\}$, where $H_k(z)$ — Hermit polynomials.

Given that function $W_0\left(x, x_0, y, y_0, \frac{\omega^2}{c^2}\right)$ is even in $\frac{\omega}{c}$, only even terms will be present in the expansion. Then:

$$W_0(x, x_0, y, y_0, \omega^2/c^2) = \sum_{k=0}^{\infty} w_{2k}(x, x_o, y, y_o) e^{-\omega^2/2c^2} H_{2k}\left(\frac{\omega}{c}\right), \text{ где}$$

$$w_{2k}(x, x_o, y, y_o) = \frac{1}{(2k)! 2^{2k} \sqrt{\pi}} \int_{-\infty}^{\infty} W_0(x, x_0, y, y_0, z^2)\, e^{-z^2/2} H_{2k}(z) dz.$$

Given the ratio:

$$\int_{-\infty}^{\infty} e^{-\omega^2/2c^2} H_{2k}\left(\frac{\omega}{c}\right) e^{-i\omega t} d\omega = 2c\sqrt{\frac{\pi}{2}} (-1)^k e^{-c^2 t^2/2} H_{2k}(ct),$$

we get:

$$W_0(x, x_0, y, y_0, t) = 2c\sqrt{\frac{\pi}{2}} \sum_{k=0}^{\infty} (-1)^k w_{2k}(x, x_o, y, y_o) e^{-c^2 t^2/2} H_{2k}(ct).$$

In this case, the solution to the original differential equation will have the form:

$$U(x, y, t) = 2c\sqrt{\frac{\pi}{2}}\, 2 \int_{-\infty}^{\infty} \sum_{k=0}^{\infty} (-1)^k w_{2k}(x, x_o(\tau), y, y_o(\tau)) e^{-c^2(t-\tau)^2/2} H_{2k}(c(t-\tau)\, d\tau.$$

Through replacing the integration variable, we get:

$$U(x,y,t) = \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_0(x, x_0(t - \frac{s\sqrt{2}}{c}), y, y_0(t - \frac{s\sqrt{2}}{c}), 2\tau^2) \frac{2\sqrt{2}(-1)^k}{(2k)!\, 2^{2k}} e^{-(s^2+\tau^2)} H_{2k}(s\sqrt{2}) H_{2k}(\tau\sqrt{2}) ds d\tau .$$

This type of solution allows us to use the Gauss-Hermite quadrature formula to calculate the integral.

To improve the convergence of the series, the Kummer method was used. Following this method, it is required to select a series whose sum is known, and the difference between the original series and the selected series should represent a rapidly converging series. As such a series, you can take:

$$U^*(x, y, t) = \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_0(x, x_0(t - {s\sqrt{2}}/{c}), y, y_0(t - {s\sqrt{2}}/{c}), q) \frac{2\sqrt{2}(-1)^k}{(2k)! 2^{2k}} e^{-(s^2+\tau^2)} H_{2k}(s\sqrt{2}) H_{2k}(\tau\sqrt{2}) ds d\tau,$$

where q — some nonnegative value.

Through integrating on variable τ and summing, we get:

$$U^*(x, y, t) = \pi c\sqrt{2} W_0(x, x_o(t), y, y_o(t), q).$$

Finally, to solve equation (1), we obtain the following expression:

$$U(x, y, t) = U^*(x, y, t) +$$

$$+\sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (W_0(x, x_0(t - {s\sqrt{2}}/{c}), y, y_0(t - {s\sqrt{2}}/{c}), 2\tau^2) - W_0(x, x_0(t - {s\sqrt{2}}/{c}), y, y_0(t - {s\sqrt{2}}/{c}), q)) \times$$

$$\times \frac{2\sqrt{2}(-1)^k}{(2k)!\, 2^{2k}} e^{-(s^2+\tau^2)} H_{2k}(s\sqrt{2}) H_{2k}(\tau\sqrt{2}) ds d\tau.$$

To sum the series, the arithmetic mean method was used. At the same time, the following was supposed: $q = 0$.

Having determined the plate deflections, it is possible to calculate the remaining components of the displacement vector and the stress tensor at any point of it using known formulas. To analyze the energy displacement of elastic waves in the plate, the Umov-Poynting energy flux density vector was calculated:

$$\bar{E} = -(\sigma_x \dot{u} + \sigma_{xy} \dot{v})\vec{i} - (\sigma_{xy} \dot{u} + \sigma_y \dot{v})\vec{j}.$$

Research Results. Calculations are carried out for the case when the force moves along a trajectory consisting of straight lines and arcs of circles (Fig. 1). The following parameter values were taken: $H = 0.25$ m; $c = 221$ m/s; $E = 232469$ N/m$^{2;}$ $\mu = 0.36$; $K = 1.864$ m^{-4}.

The parameters of the law of motion of the force along the trajectory were selected in such a way that at the time under consideration, the force was always at the same point of the trajectory marked with an asterisk, having different values of speed v and acceleration a, as well as at different values of the radius of trajectory R_2. To study the stress-strain state of the plate, displacements and stresses near the point of application of force were calculated.

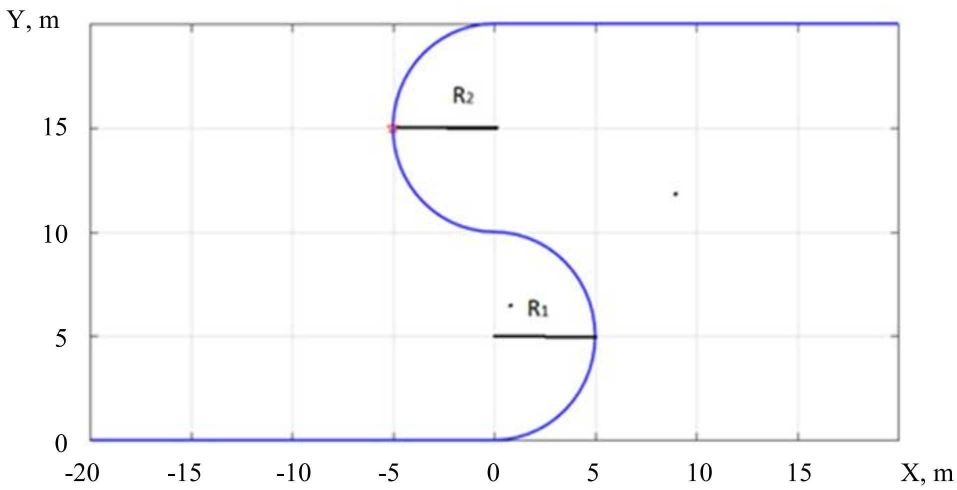

Fig. 1. Trajectory of the concentrated force

Figures 2 and 3 show the change in displacements and stresses during the movement of a concentrated force along a given trajectory at $v = 25$ m/s, $a = 0$ m/s^2, $R_2 = 5$ m. The change of these values along the Y-axis does not practically differ from their change along the X-axis.

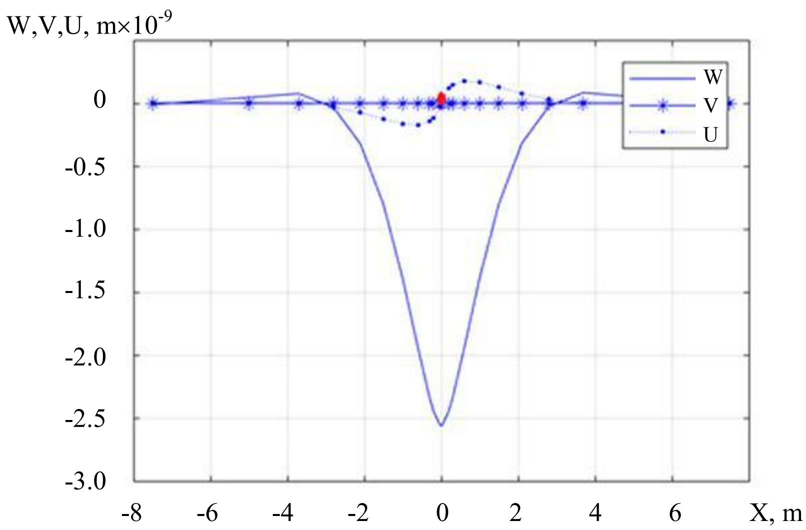

Fig. 2. Change in displacements: W — vertical; U — along the X axis; V — along the Y axis

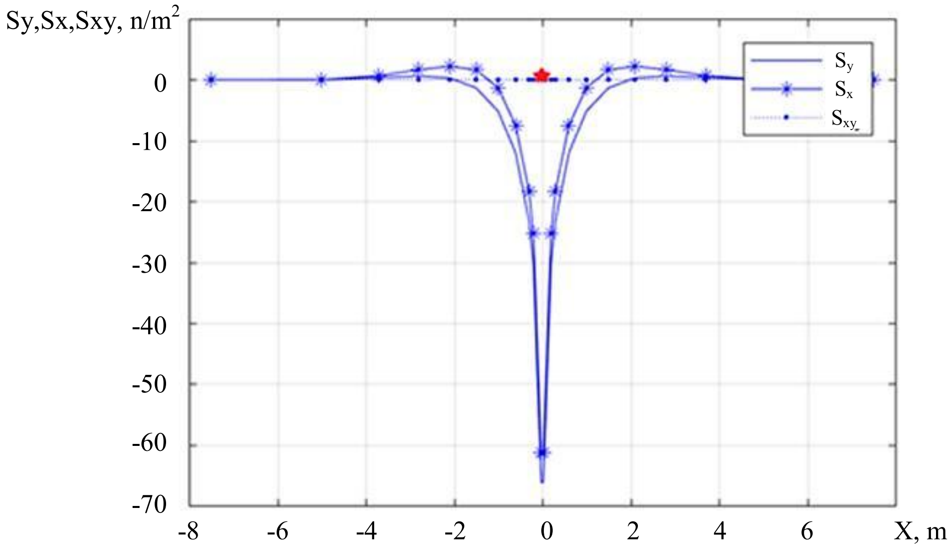

Fig. 3. Change in voltages: W — vertical; U — along the X axis; V — along the Y axis

Figure 4 shows the movement of the energy of elastic waves near the concentrated force, whose position on the trajectory is marked with a red dot. The vectors determine the amount and direction of energy transfer at a given point.

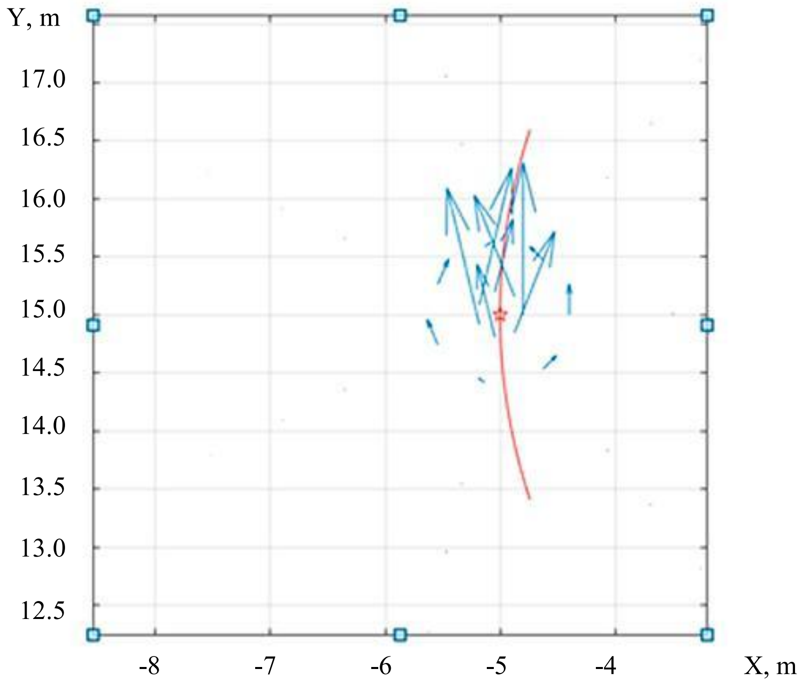

Fig. 4. Energy flux density vector at v = 25 m/s, a = 0 m/s^2, R_2 = 5 m

The calculations have shown that with an increase in the speed of the force movement, there is no qualitative change in displacements and stresses, but only their quantitative growth occurs. A slight change in the qualitative behavior of displacements and stresses is observed only at sufficiently high speeds, when the condition $v > c$ is met. This follows from Fig. 5, 6 (a = 0 m/sc^2, R_2 = 5 m).

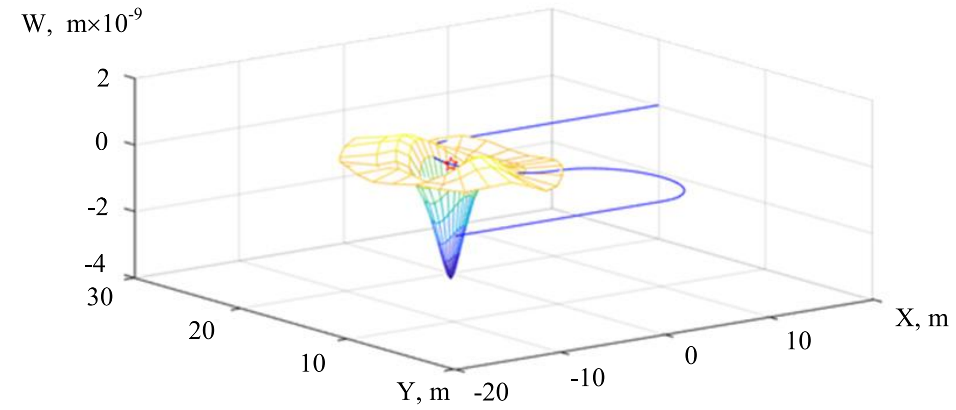

Fig. 5. Changing vertical movements at v = 275 m/s

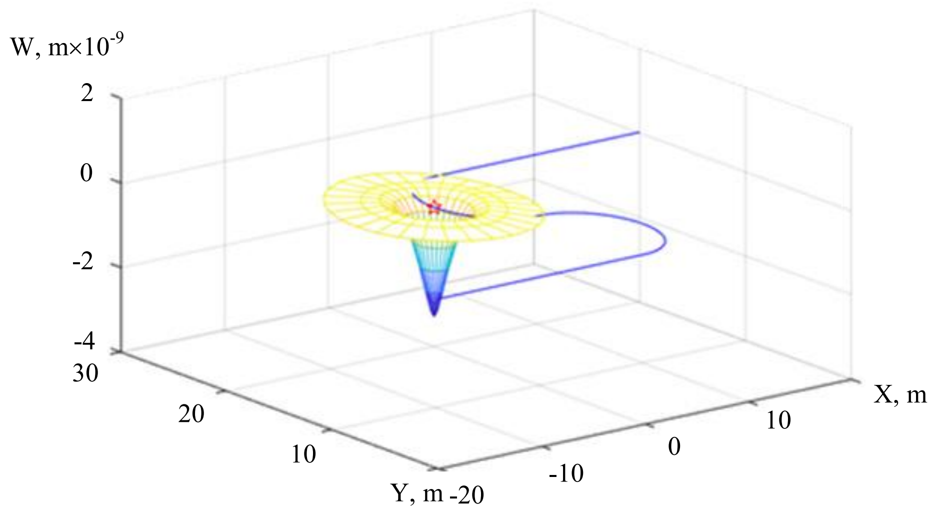

Fig. 6. Changing vertical movements at v = 75 m/s

Figures 7 and 8 show the change in the maximum vertical displacements W and stresses S_x , S_y depending on the speed of the force movement at $a = 0$ m/s^2, $R_2 = 5$ m. The remaining components of displacements and stresses assumed sufficiently low values and therefore were not of constructive interest when analyzing the stress-strain state of the plate.

Calculations performed at different values of acceleration and radius R_2, have shown that these factors have little effect on the stress-strain state of the plate. The qualitative picture of wave energy propagation near the concentrated force also weakly depends on these factors.

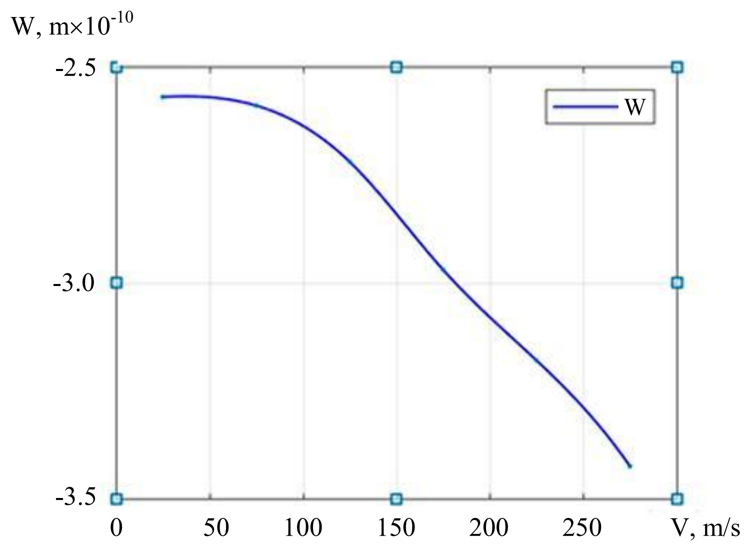

Fig. 7. Dependence of maximum vertical displacements on concentrated force movement speed

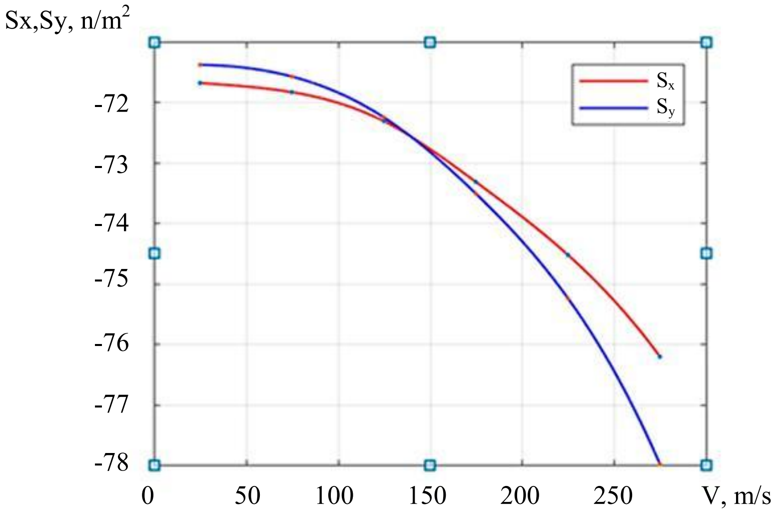

Fig. 8. Dependence of maximum voltages on concentrated force movement speed

Discussion and Conclusions. The most significant effect on the stress-strain state of the plate and the propagation of elastic wave energy near the concentrated force is exerted by the speed of its movement. The radius of curvature of the trajectory and the acceleration of the force movement do not significantly affect.

The calculation results indicate that the method of solving problems on the action of a moving load is quite stable within a wide range of changes in the speed of its movement. The method is economical and simple, because it uses already known fundamental solutions.

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